In Neil Salkind (Ed.), *Encyclopedia of Research Design.*
Thousand Oaks, CA: Sage. 2010

Guttman Scaling

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1 Introduction

Guttman scaling was developed by Louis Guttman (1944, 1950) and was first used as part of the classic work on the *American Soldier*. Guttman scaling is applied to a set of binary questions answered by a set of subjects. The goal of the analysis is to derive a single dimension that can be used to position both the questions and the subjects. The position of the questions and subjects on the dimension can then be used to give them a numerical value. Guttman scaling is used in social psychology and in education.

2 An example of a perfect Guttman scale

Suppose that we test a set of Children and that we assess their mastery of the following types of mathematical concepts: 1) counting from 1 to 50, 2) solving addition problems, 3) solving subtraction problems, 4) solving multiplication problems, and 5) solving division problems.

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Table 1: The pattern of responses of a perfect Guttman scale. A value of 1 means that the child (row) has mastered the type of problem (column), a value of 0 means that the child does not master the type of problem.

	Problems				
Children	Counting	Addition	Subtraction	Multiplication	Division
S_1	1	0	0	0	0
S_2	1	1	0	0	0
S_3	1	1	1	0	0
S_4	1	1	1	1	0
S_5	1	1	1	1	1

Some children will be unable to master any of these problems, and these children do no provide information about the problems so we will not consider them. Some children will master counting but nothing more, some will master addition and we expect them to have mastered addition but no other concepts; some children will master subtraction and we expect them to have mastered counting and addition; some children will master multiplication and we expect them to have mastered subtraction, addition, and counting. Finally, some children will master division problem and we expect them to have mastered counting, addition, subtraction, and multiplication. What we do *not* expect to find, however, are children, for example, who have mastered division but who have not mastered addition or subtraction or multiplication. So the set of patterns of responses that we expect to find is well structured and is shown in Table 1. The pattern of data displayed in this figure is consistent with the existence a single dimension of mathematical ability. In this framework, a child has reached a certain level of this mathematical ability and can solve all the problems below this level and none of the problems above this level.

When the data follow the pattern illustrated in Table 1, the rows and the columns of the table can both be represented on a single dimension. The operations will be ordered from the easiest to the hardest and a child will be positioned on the right of the most difficult type of operation solved. So the data from Table 1 can be represented by the following order:

$$\text{Counting} \quad S_1 \quad \text{Addition} \quad S_2 \quad \text{Subtraction} \quad S_3 \quad \text{Multiplication} \quad S_4 \quad \text{Division} \quad S_5 \,. \tag{1}$$

This order can be transformed into a set of numerical values by assigning numbers with equal steps between two contiguous points. For example, this set of numbers can represent the numerical values corresponding to Table 1:

Counting	S_1	Addition	S_2	Subtraction	S_3	Multiplication	S_4	Division	S_5
1	2	3	4	5	6	7	8	9	10

This scoring scheme implies that the score of an observation (*i.e.,* a row in Table 1) is proportional to the number of non-zero variables (*i.e.,* columns in Table 1) for this row.

The previous quantifying scheme assumes that the differences in difficulty are the same between all pairs of contiguous operations. In real applications, it is likely that these differences are not the same. In this case, a way of estimating the size of the difference between two contiguous operations is to consider that this difference is *inversely* proportional to the number of children who solved a given operation (*i.e.,* an easy operation is solved by a large number of children, a hard one is solved by a small number of children).

2.1 How to order the rows of a matrix to find the scale

When the Guttman model is valid, there are multiple ways of finding the correct order of the rows and the columns which will give the format of the data as presented in Table 1. The simplest approach is to re-order rows and columns according to their marginal sum. Another theoretically interesting procedure is to use correspondence analysis (which is a type of factor analysis tailored for qualitative data) on the data table, then the coordinates on the first factor of the analysis will provide the correct ordering of the rows and the columns.

3 Imperfect scale

In practice, it is rare to obtain data that perfectly fit a Guttman scaling model. When the data do not conform to the model, one approach is to relax the uni-dimensionality assumption and to assume that the underlying model involves several dimensions. Then, these dimensions can be obtained and analyzed with multidimensional techniques such as correspondence analysis (which can be seen as a multidimensional generalization of Guttman scaling) or multidimensional scaling. Another approach is to consider that the deviations from the ideal scale are random errors. In this case, the problem is to recover the Guttman scale from noisy data. There are several possible ways to fit a Guttman scale to a set a data. The simplest method (called the Goodenough-Edwards method) is to order the rows and the columns according to their marginal sum. An example of a set of data corresponding to such an imperfect scale is given in Table 2. In this table the "errors" are indicated with a *, and there are three of them. This number of errors can be used to compute a coefficient of reproducibility denoted C_R and defined as

$$C_R = 1 - \frac{\text{Number of errors}}{\text{Number of possible errors}} \ . \tag{2}$$

The number of possible errors is equal to the number of entries in the data table which is equal to the product of the numbers of rows and columns of this table. For the data in Table 2, there are three errors out of $5 \times 6 = 30$ possible errors, this gives a value of the coefficient of reproducibility equal to

$$C_R = 1 - \frac{3}{30} = .90 \ . \tag{3}$$

According to Guttman, a scale is acceptable if it contains less than 10% of erroneous entry which is equivalent to consider that a scale is acceptable if the value of its C_R is equal to or larger than .90. In practice, it is often possible to improve the C_R of a scale by eliminating rows or columns which contain a large proportion of errors. Unfortunately, this practice may also lead to capitalize on random errors and may give and unduly optimistic view of the actual reproducibility of a scale.

Table 2: An imperfect Guttman scale. Values with a * are considered errors. Compare with Table 1 showing a perfect scale.

	Problems					
Children	Counting	Addition	Subtraction	Multiplication	Division	Sum
C_1	1	0	0	0	0	1
C_2	1	0*	1*	0	0	2
C_3	1	1	1	0	0	3
C_4	1	1	0*	1	0	3
C_5	1	1	1	1	1	5
Sum	5	3	3	2	1	—

Related entries

Canonical correlation analysis, correspondence analysis, categorical variables, Likert scaling, Principal component analysis, Thurstone scaling.

Further readings

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