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How to Begin a New Topic in Mathematics: Does It Matter to Students' Performance in Mathematics?

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HOW TO BEGIN A NEW TOPIC IN MATHEMATICS

Does It Matter to Students' Performance in Mathematics?

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The authors use Canadian data from the Third International Mathematics and Science Study to examine six instructional methods that mathematics teachers use to introduce new topics in mathematics on performance of eighth-grade students in six mathematical areas (mathematics as a whole, algebra, data analysis, fraction, geometry, and measurement). Results of multilevel analysis with students nested within schools show that the instructional methods of having the teacher explain the rules and definitions and looking at the textbook while the teacher talks about it had little instructional effects on student performance in any mathematical area. In contrast, the instructional method in which teachers try to solve an example related to the new topic was effective in promoting student performance across all mathematical areas.

Keywords: *instructional methods; instructional effects; new topics; mathematics achievement*

It is well recognized that the way that mathematics teachers instruct mathematics matters to the learning of students in mathematics. Brown and Borko (1992, p. 212) view instruction as "the process of facilitating students' comprehension," which "consists of a variety of teaching acts, such as organizing and managing the classroom, presenting clear explanations, and providing for student practice." The beginning of a new topic in mathematics marks the starting point of this instructional process. In this study, we examined the way that mathematics teachers introduce new topics in mathematics as it relates to mathematics performance of their students. We asked three research questions. The first research question pertained to whether different instructional methods that mathematics teachers use to introduce new topics in mathematics affect mathematics performance of their students. We answered this

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research question by examining whether mathematics teachers who employ different instructional methods to introduce new topics in mathematics have students who perform at different levels in mathematics achievement.

Based on our belief on differential teacher effectiveness, we formulated the second research question as whether there is a significant variation in terms of teachers' instructional effects on mathematics performance of their students. To answer this research question, we examined whether mathematics teachers who use the same instructional method to introduce new topics in mathematics have students who perform at different levels in mathematics achievement. Finally, we believe that the instructional practice of mathematics teachers as a whole in a school has influences on mathematics performance of students within the school (often referred to as "school effects"). We examined whether the average instructional practice of mathematics teachers within a school (i.e., the extent to which mathematics teachers in a school practice the same instructional method to introduce new topics in mathematics) has any significant impact on mathematics performance of students within the school. This was our third research question pertaining to whether there is a significant school mean instructional effect on mathematics performance of students.

The motive to undertake these research questions was our serious dissatisfaction with the research literature in which researchers have paid little attention to the issue of effective ways to introduce new topics in mathematics, effective in terms of facilitating and promoting students' mastery of mathematical topics to be learned and, as a result, students' performance in mathematics. So far, the decision of mathematics teachers in terms of how to begin a new topic in mathematics is largely based on traditional wisdom or instructional convenience rather than working knowledge derived from empirical evidence of research studies. We perceive a great potential in using this issue to influence the instructional process of mathematics teachers for better mathematics performance of students.

REVIEW OF LITERATURE

MATHEMATICS INSTRUCTION AND PERFORMANCE

The National Commission on Mathematics and Science Teaching for the 21st Century (2000) emphasizes that the quality of classroom instruction is key to the quality of students' learning. Although the research literature is barren on effective ways to introduce new topics in mathematics, there are different instructional methods for teaching mathematics. These general instructional methods can be "carried over" as ways to begin a new topic in mathematics. In this sense, they are relevant to this study and provide clues for understanding instructional effects of different ways to introduce new topics in mathematics.

Secada (1992) classified instruction in mathematics into direct instruction, continuous progress, individualized instruction, and cognitively guided instruction. "Direct instruction is a highly structured form of teacher behaviors that are thought to support student engagement in and learning of mathematics" (Secada, 1992, p. 649). This instructional format carefully sequences teacher behaviors that guide students to learn mathematics in a structural manner. Mathematics teachers who are effective in using direct instruction are "active, well organized, and strongly academically oriented. They emphasized whole class instruction . . . managed their classrooms effectively . . . [and] asked many questions during class discussion" (Everston, Anderson, Anderson, & Brophy, 1980, p. 58, cited in Secada, 1992, p. 649).

"Continuous progress includes direct instruction as one of its features; in addition, students are to progress through a well-specified hierarchy of skills, and they should be grouped on the basis of their ongoing progress through that curriculum" (Secada, 1992, pp. 648-649). This instructional format links closely teacher behaviors with student outcomes (mathematical knowledge and skills) to ensure progress of students in the learning of mathematics. Grouping of students is used in this instructional format to clearly define teacher behaviors and student outcomes.

Individualized instruction recognizes individual differences in the need and ability to learn mathematics. There is also the recognition that students learn mathematics in uniquely different ways. Individualized instruction therefore emphasizes that instructional methods need to be different from student to student to facilitate the learning of mathematics for all students. Based on the knowledge of mathematics teachers in terms of a student's need, ability, and (unique) way to learn mathematics, mathematics teachers tailor various instructional plans for individual students, with a commitment to individual growth and development in the learning of mathematics.

Cognitively guided instruction (CGI) is

> based on four interlocking principles: (a) teacher knowledge of how mathematical content is learned by their students, (b) problem solving as the focus of instruction, (c) teacher access to how students are thinking about specific problems, and (d) teacher decision-making based on teachers knowing how their students are thinking. (Secada, 1992, p. 649)

Unlike direct instruction, CGI does not regulate instructional behaviors of mathematics teachers directly; instead, it allows mathematics teachers the flexibility to engage students in the learning of mathematics on the basis of their knowledge about the thinking process of their students.

Different versions of these four basic instructional formats have been widely practiced in mathematics classrooms. Do these different instructional formats result in different mathematics performance among students? The Third International Mathematics and Science Study (TIMSS) has a video component that looks in detail into instructional practices of mathematics teachers, particularly in high-achieving countries. Hiebert et al. (2003) reported the TIMSS 1999 video study. Although all countries are similar in the way in which mathematics lessons are structured (whole-class instruction or discussion is present in most mathematics lessons across all countries, with occasional attentions to individualized student work), mathematics teachers in high-achieving countries (i.e., Australia, Czech Republic, Hong Kong, Japan, the Netherlands, and Switzerland) demonstrate unique ways of teaching mathematics. First, although most classroom time is devoted to problem solving across all countries, much more problems are presented in mathematical symbols rather than real-life contexts in high-achieving countries. Second, mathematics teachers in high-achieving countries spend more time working on new content than reviewing old content and pay close attention to the conceptual development of students in mathematics. One of the conclusions made by Hiebert et al. is that there are effective methods of teaching mathematics that support high achievement in mathematics.

THEORETICAL PERSPECTIVES AND FRAMEWORKS

The research literature has witnessed attempts to use theoretical approaches to analyze and understand mathematics teachers' instructional practices as they relate to students' learning of mathematics. These theories are particularly relevant to our study. As a survey study, our analysis detected differential effects of the ways in which mathematics teachers introduce new topics in mathematics. Although we addressed the "whether" question, our data at hand are not equipped to address the "how" question (the mechanism that different ways of introducing new topics function to promote students' learning of mathematics). As supplementary efforts, these existing theories were explored to offer some insights into the "how" question.

Serafino and Cicchelli (2003) demonstrated how constructivist theories can be employed to analyze different instructional formats. Constructivist theories emphasize the cognitive process of learning. These theories are particularly important to our study in that constructivists emphasize the connection between prior and new knowledge. Constructivists carefully design appropriate learning activities (e.g., exploring, applying, discovering, and reasoning) to promote active engagement of students in the construction of their own knowledge. A key of the constructivist approach to mathematics instruction is to create a problem-rich environment that facilitates the construction of various connections between prior and new knowledge. Many mathematics teachers introduce new topics by working with students on problems that both refresh students' prior knowledge and foreshadow the new knowledge that students are expected to learn as a way to establish connections between prior and new knowledge.

Theories of peer learning have been used to support grouping practice (or cooperative learning) in mathematics instruction (e.g., Chapman 1995). These theories emphasize the importance of knowledge sharing in promoting students' learning of mathematics. Hoffman (2002) examined various theoretical perspectives to explain the power of peer learning. Social-motivational perspectives believe that student choice is essential to support autonomous learning and that opportunities to choose are rewarding and motivational. Piagetian perspectives emphasize that common interest is the contextual condition for learning and conceptual development. Vygotskian perspectives believe that differences in ability translate into learning. Such a transfer can happen easily in a knowledge-sharing context in which mentoring and tutoring occur frequently among students.

Finally, the modeling theory offers a useful way to understand why some innovative instructional formats in mathematics fail to produce desirable outcomes in students' learning. Schorr and Koellner-Clark (2003) believe that all teachers have their own models for teaching and learning mathematics, formed on a series of teaching experiences in mathematics. "When teachers adopt specific changes or strategies (like using manipulatives) into their classroom practice, they often do so within the framework of their older (more traditional) models" (p. 198). That is, "the new technique or strategy is added onto their model, without fundamentally changing their worldview of what mathematics instruction is or should be" (p. 198). Because the nature of their instructional practices remains relatively unchanged, the adoption of a new technique or strategy often produces superficial effects that can hardly affect students' learning.

In our study, theoretical perspectives as reviewed above can be reasonably matched onto instructional practices that mathematics teachers used to introduce new topics in mathematics. The differential students' mathematics performance as a result of mathematics teachers' adopting different instructional formats to introduce new topics in mathematics may then be understood in a theoretical manner. Such efforts partially unearth the mechanism of differential instructional effects on students' mathematics performance.

CAUSAL DIRECTION OF MATHEMATICS INSTRUCTION AND PERFORMANCE

Is mathematics performance a result of mathematics instruction or is the selection of instruction a result of the assessment of performance? The research literature appears to specify mathematics performance as a result of mathematics instruction. Every instructional method is originally designed to help all students. The most obvious example is cooperative learning that is designed precisely to benefit all students (see Johnson, Johnson, & Holubec, 1987) and does benefit all students (see Vaughan, 2002). For another example, based on a research synthesis of 108 studies, Kulik, Kulik, and Bangert-Drowns (1990) reported that Bloom's mastery learning instructional approach increases academic performance of all students (for all groups).

Although mathematics teachers do consider students' mathematics abilities when they decide on instructional methods, their decision usually takes the form of modifications. Secada (1992, p. 649) stated that "instruction that works for all students can be modified to work for low-achieving students from various backgrounds." He presented examples of such modifications as "more highly structured lessons, greater attention to basic skills, [and] deeper coverage of less content to ensure mastery" (p. 649). Lefrancois (2000) also emphasized the individualization of each instructional method for adaptation to the needs, interests, and abilities of all students. Therefore, the fundamental educational principles underlining each instructional method work for all students, with the flexibility of implementation modifications (e.g., different cognitive emphases, different time allocations) as a way to tap into the maximum potential of each student.

METHOD

DATA

The student questionnaire used in the TIMSS contains a scale that measured the different instructional practices that mathematics teachers employed to introduce new topics in mathematics. This made TIMSS data suitable to this study. Specifically, we used the Canadian sample from the latest data from the TIMSS-Repeated (TIMSS-R). There is no federal department of education in Canada (i.e., education is a provincial jurisdiction). Without (unified) national curricular and instructional standards, it is likely that different instructional formats are practiced in mathematics in a greater degree in Canada. For this reason, we considered Canadian data as ideal to examine instructional effects on mathematics performance.

The target population was students who enrolled in the eight grade in the 1998-1999 school year. The TIMSS-R employed a stratified sampling procedure in which schools were first selected. One class in the eighth grade was then selected from each sampled school, and all students in the selected class participated in the TIMSS-R. Sampled students had an average age of 14 years. Participating students took achievement tests in mathematics and science and completed questionnaires on home and school experiences related to the learning of mathematics and science. Characteristics of social environment, particularly those at home and in school, tend to be most influential on students at this age. School administrators and teachers also completed questionnaires regarding school operations and classroom practices. As mentioned earlier, our analysis focused on the Canadian sample that included 8770 students from 385 schools.

VARIABLES

Dependent variables were mathematics performance in our analysis, coming from achievement test in mathematics. We considered student performance in different mathematical areas as classified in the TIMSS-R, including (a) mathematics as a whole, (b) algebra, (c) data analysis, (d) fraction, (e) geometry, and (f) measurement. Major independent variables were instructional methods to begin a new topic in mathematics, coming from the student questionnaire. These instructional methods included (a) having the teacher explain the rules and definitions, (b) discussing a practical or story problem related to everyday life, (c) working together in pairs or small groups on a problem project, (d) having the teacher ask students what they know related to the new topic, (e) looking at the textbook while the teacher talks about it, and (f) trying to solve an example related to the new topic. We examined these instructional methods separately. Each instructional method was measured in terms of the frequency that mathematics teachers used to introduce new topics in mathematics (almost always, pretty often, once in a while, and never). Because there were four frequency categories, we created three dichotomous variables to represent each instructional method with "never" as the reference (almost always versus never, pretty often versus never, and once in a while versus never).

Other independent variables came from student and school questionnaires. We selected gender, age, mother's education, father's education, immigration status, mother's immigration status, and father's immigration status to describe individual and family characteristics and used class size, school male enrollment, school female enrollment, school location, school mean mother's education, and school mean father's education to describe school characteristics. Gender, immigration status, mother's immigration status, and father's immigration status were coded as dichotomous variables. Other student characteristics were continuous variables. Class size, school male enrollment, and school female enrollment were simply numbers of students. School location was coded into a series of dichotomous variables. School mean mother's education and school mean father's education were aggregated from the student level to the school level. Note that these independent variables were used mainly as control variables to derive "purer" teacher instructional effect on student performance in various mathematical areas.

Another group of school-level variables measured the average extent to which teachers practiced each instructional method in a school. These school-level variables were labeled as school mean or average instructional effects, and each was constructed by aggregating within each school the three dichotomous variables representing a particular instructional method. These school-level variables then represented school mean instructional effects on academic achievement over and above teacher (individual) instructional effects. For the purpose of data analysis, all continuous student-level variables were standardized to have a mean of 0 and a standard deviation of 1, and all continuous school-level variables were centered among their grand means.

ANALYSIS

Statistically, multilevel analysis techniques were used to address our research hypotheses (see Raudenbush and Bryk 2002). We developed a series of two-level models with students (Level 1) nested within schools (Level 2), performing separate analyses for various mathematical areas and instructional methods. Because one class in the eighth grade was randomly drawn to represent each sampled school in the TIMSS-R, there was no meaningful hierarchy between classes and schools. We specified our multilevel model as follows:

$$\text{Level 1 model: } Y_{ij} = \beta_{0j} + \beta_{1j}\, Method_{ij} + \sum_{p=2}^{m} X_{pij} + \varepsilon_{ij}.$$

$$\text{Level 2 model: } \beta_{0j} = \gamma_{00} + \gamma_{01}\, MeanMethod_{0j} + \sum_{q=2}^{n} W_{0qj} + u_{0j},\ \beta_{1j} = \gamma_{10} + u_{1j}.$$

The Level 1 model regressed academic achievement (Y_{ij}) on each instructional method (*Method*) in the presence of statistically significant student-level variables (X_{pij}). The coefficient, β_{1j}, measured the effect of an instructional method on academic achievement. We also allowed this effect (β_{1j}) to vary at the school level to examine whether teacher instructional effect varied across schools (u_{1j}). ε_{ij} is the Level 1 error term assumed to have a normal distribution with a mean of zero and variance (i.e., the Level 1 variance). The Level 2 model then focused on school mean instructional effect over and above teacher (individual) instructional effect (associated with a particular instructional method) with adjustment for school-level variables. In other words, each school mean instructional effect (γ_{01}, the coefficient of *MeanMethod*) was estimated in the presence of statistically significant school-level variables (W_{qj}). u_{0j} and u_{1j} are Level 2 error terms assumed to have a multivariate normal distribution with a mean of zero as well as Level 2 variance and covariance components (τ_{00}, τ_{11}, and τ_{01}).

We selected student-level and school-level predictors of mathematics performance based on a theoretical scheme on school effects in the literature of school effectiveness. Willms (1992, p. 31) discussed the "stronger models" for monitoring schooling outcomes, emphasizing student inputs (student and family characteristics) as key student-level predictors and ecology and milieu (school context and climate) as key school-level predictors. Although the TIMSS data did not include measures on all key components of the stronger models, we located a sizeable number of variables relevant to these models. In general, the TIMSS data had a much stronger match of key predictors at the student level than at the school level. Willms (1992) also implied that it is conventional to model student academic performance with a linear function form especially when performance is measured at only one time point. We adopted this tradition for simplicity given the absence of stronger theoretical arguments for more complicated forms of the model (e.g., nonlinear function form).

We performed multilevel analysis on the PC platform of the HLM program (Raudenbush et al. 2004). The HLM provided residual files (discrepancies between the observed and fitted values) at both levels, which allowed us to check the tenability of statistical assumptions underlying our multilevel models. As recommended in Raudenbush et al. (2004, pp. 36-46), at the student level, we used the Q-Q plot to examine the normality of Level 1 errors, and we examined the relationship between ordinary least-square residuals and empirical Bayes estimates to ensure the adequacy of the fitted models at the school level. Overall, the results of multilevel diagnoses were acceptable to us.

RESULTS

We analyzed the effect of each instructional method to begin a new topic in mathematics on academic achievement of students in a number of mathematical areas. Table 1 presents effects of different instructional methods on academic achievement in mathematics overall. Having the teacher explain the rules and definitions turned out to have no impact on overall achievement in mathematics at either student or school level. Unexpectedly, discussing a practical or story problem related to everyday life demonstrated negative effects on overall achievement when mathematics teachers used this instructional method almost always or pretty often to introduce new topics in mathematics. In contrast, this instructional method had a positive effect when teachers used it once in a while to introduce new topics.

Working together in pairs or small groups on a problem project demonstrated a negative effect on overall achievement in mathematics when mathematics teachers relied on it almost always to introduce new topics in mathematics. Having the teacher ask students what they know related to the new topic had a negative effect when teachers used it almost always to introduce new topics but a positive effect when teachers used it once in a while to introduce new topics. Students in schools where a higher percentage of teachers used this instructional method pretty often to introduce new topics showed lower overall achievement in mathematics.

Looking at the textbook while the teacher talks about it showed a negative effect on overall achievement in mathematics when mathematics teachers relied on it almost always to introduce new topics in mathematics but a positive effect when teachers used it pretty often to introduce new topics. Students in schools where a higher percentage of teachers almost always introduced new topics by trying to solve an example related to the new topic showed higher overall achievement. Finally, there was no variation in teacher instructional effect across schools associated with any instructional method.

Results on effects of different instructional methods to begin a new topic in mathematics on academic achievement in other mathematical areas are presented in Tables 2 through 6. With our interpretation about Table 1 as an illustration, results in Tables 2 through 6 could be interpreted in a similar manner. In many cases, instructional methods showed similar effects on

(text continues on p. 473)

TABLE 1: Different Instructional Methods of Beginning a New Topic in Mathematics on Student Performance in Mathematics

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Having the teacher explain the rules and definitions			
Almost always (vs. never)	–0.037	0.005	0.054
Pretty often (vs. never)	0.044	0.006	–0.174
Once in a while (vs. never)	0.029	0.003	0.349
Discussing a practical or story problem related to everyday life			
Almost always (vs. never)	–0.236*	0.001	1.023
Pretty often (vs. never)	–0.115*	0.002	–0.507
Once in a while (vs. never)	0.170*	0.003	–0.079
Working together in pairs or small groups on a problem project			
Almost always (vs. never)	-0.377*	0.005	0.440
Pretty often (vs. never)	–0.057	0.021	0.293
Once in a while (vs. never)	0.048	0.003	–0.338
Having the teacher ask us what we know related to the new topic			
Almost always (vs. never)	–0.149*	0.029	0.052
Pretty often (vs. never)	–0.026	0.002	–0.817*
Once in a while (vs. never)	0.119*	0.002	0.615

(continued)

TABLE 1 (continued)

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Looking at the textbook while the teacher talks about it			
Almost always (vs. never)	–0.129*	0.013	0.518
Pretty often (vs. never)	0.079*	0.002	–0.334
Once in a while (vs. never)	0.009	0.002	–0.496
Trying to solve an example related to the new topic			
Almost always (vs. never)	0.021	0.002	0.820*
Pretty often (vs. never)	–0.047	0.002	–0.544
Once in a while (vs. never)	0.074	0.002	–1.034

NOTE: Teacher instructional effect measures the effect of an instructional method on academic achievement, estimated in the presence of age, mother's education, and father's education as statistically significant covariates. Variation in teacher instructional effect measures the extent to which teacher instructional effect varies across schools. School mean instructional effect measures whether the average degree to which teachers practice an instructional method in a school influences student academic achievement, estimated in the presence of school mean mother's education and school female enrollment (1 unit = 100 students) as statistically significant covariates.
*$p < .05$.

TABLE 2: Different Instructional Methods of Beginning a New Topic in Mathematics on Student Performance in Algebra

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Having the teacher explain the rules and definitions			
Almost always (vs. never)	–0.018	0.003	–0.056
Pretty often (vs. never)	0.033	0.004	–0.170
Once in a while (vs. never)	0.012	0.008	0.567
Discussing a practical or story problem related to everyday life			
Almost always (vs. never)	–0.222*	0.005	1.049
Pretty often (vs. never)	–0.079	0.004	–0.573
Once in a while (vs. never)	0.147*	0.018	–0.021
Working together in pairs or small groups on a problem project			
Almost always (vs. never)	–0.336*	0.008	0.745
Pretty often (vs. never)	–0.094	0.013	0.403
Once in a while (vs. never)	0.068*	0.004	–0.406
Having the teacher ask us what we know related to the new topic			
Almost always (vs. never)	–0.182*	0.010	0.254
Pretty often (vs. never)	0.031	0.001	–0.898*
Once in a while (vs. never)	0.103*	0.001	0.564

(continued)

TABLE 2 (continued)

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Looking at the textbook while the teacher talks about it			
Almost always (vs. never)	–0.045	0.009	0.427
Pretty often (vs. never)	0.032	0.008	–0.323
Once in a while (vs. never)	–0.058	0.002	–0.562
Trying to solve an example related to the new topic			
Almost always (vs. never)	0.045	0.002	0.762*
Pretty often (vs. never)	–0.048	0.007	–0.634
Once in a while (vs. never)	0.013	0.004	–0.823

NOTE: Teacher instructional effect measures the effect of an instructional method on academic achievement, estimated in the presence of age, mother's education, and father's education as statistically significant covariates. Variation in teacher instructional effect measures the extent to which teacher instructional effect varies across schools. School mean instructional effect measures whether the average degree to which teachers practice an instructional method in a school influences student academic achievement, estimated in the presence of school mean mother's education and school female enrollment (1 unit = 100 students) as statistically significant covariates.
$*p < .05$.

TABLE 3: Different Instructional Methods of Beginning a New Topic in Mathematics on Student Performance in Data Analysis

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Having the teacher explain the rules and definitions			
Almost always (vs. never)	–0.014	0.002	0.015
Pretty often (vs. never)	0.058	0.006	–0.068
Once in a while (vs. never)	–0.013	0.027*	0.211
Discussing a practical or story problem related to everyday life			
Almost always (vs. never)	–0.253*	0.028	0.661
Pretty often (vs. never)	–0.148*	0.002	–0.481
Once in a while (vs. never)	0.208*	0.005	0.150
Working together in pairs or small groups on a problem project			
Almost always (vs. never)	–0.207*	0.014	–0.071
Pretty often (vs. never)	–0.082	0.005	0.152
Once in a while (vs. never)	–0.009	0.052	0.067
Having the teacher ask us what we know related to the new topic			
Almost always (vs. never)	–0.230*	0.004	0.067
Pretty often (vs. never)	0.017	0.002	–0.695*
Once in a while (vs. never)	0.109*	0.005	0.539*

(continued)

TABLE 3 (continued)

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Looking at the textbook while the teacher talks about it			
Almost always (vs. never)	–0.203*	0.021	0.306
Pretty often (vs. never)	0.124*	0.013	–0.031
Once in a while (vs. never)	0.020	0.010	–0.152
Trying to solve an example related to the new topic			
Almost always (vs. never)	–0.016	0.002	0.568*
Pretty often (vs. never)	–0.018	0.002	–0.319
Once in a while (vs. never)	0.091	0.003	–0.577

NOTE: Teacher instructional effect measures the effect of an instructional method on academic achievement, estimated in the presence of age, gender, immigration status, mother's immigration status, and father's education as statistically significant covariates. Variation in teacher instructional effect measures the extent to which teacher instructional effect varies across schools. School mean instructional effect measures whether the average degree to which teachers practice an instructional method in a school influences student academic achievement, estimated in the presence of school mean mother's education and school female enrollment (one unit = 100 students) as statistically significant covariates.
*$p < .05$.

TABLE 4: Different Instructional Methods of Beginning a New Topic in Mathematics on Student Performance in Fraction

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Having the teacher explain the rules and definitions			
Almost always (vs. never)	–0.047	0.008	–0.101
Pretty often (vs. never)	0.046	0.022*	–0.071
Once in a while (vs. never)	0.015	0.003	0.683
Discussing a practical or story problem related to everyday life			
Almost always (vs. never)	–0.349*	0.019	0.928
Pretty often (vs. never)	–0.100*	0.008	–0.768
Once in a while (vs. never)	0.212*	0.014	0.089
Working together in pairs or small groups on a problem project			
Almost always (vs. never)	–0.420*	0.004	0.262
Pretty often (vs. never)	–0.087	0.007	0.210
Once in a while (vs. never)	–0.084*	0.013	–0.275
Having the teacher ask us what we know related to the new topic			
Almost always (vs. never)	–0.251*	0.019	0.121
Pretty often (vs. never)	–0.007	0.002	–0.827*
Once in a while (vs. never)	0.180*	0.003	0.574

(continued)

TABLE 4 (continued)

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Looking at the textbook while the teacher talks about it			
Almost always (vs. never)	–0.183*	0.026*	0.432
Pretty often (vs. never)	0.103*	0.008	–0.247
Once in a while (vs. never)	0.042	0.004	–0.315
Trying to solve an example related to the new topic			
Almost always (vs. never)	–0.006	0.005	0.763*
Pretty often (vs. never)	–0.046	0.002	–0.626
Once in a while (vs. never)	0.074	0.013	–0.766

NOTE: Teacher instructional effect measures the effect of an instructional method on academic achievement, estimated in the presence of age, gender, immigration status, mother's immigration status, mother's education, and father's education as statistically significant covariates. Variation in teacher instructional effect measures the extent to which teacher instructional effect varies across schools. School mean instructional effect measures whether the average degree to which teachers practice an instructional method in a school influences student academic achievement, estimated in the presence of school mean mother's education and school female enrollment (1 unit = 100 students) as statistically significant covariates.
*$p < .05$.

TABLE 5: Different Instructional Methods of Beginning a New Topic in Mathematics on Student Performance in Geometry

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Having the teacher explain the rules and definitions			
Almost always (vs. never)	–0.007	0.008	–0.208
Pretty often (vs. never)	–0.065*	0.002	0.199
Once in a while (vs. never)	0.181*	0.003	0.307
Discussing a practical or story problem related to everyday life			
Almost always (vs. never)	–0.240*	0.002	1.270*
Pretty often (vs. never)	–0.117*	0.003	–0.561
Once in a while (vs. never)	0.090*	0.014	0.276
Working together in pairs or small groups on a problem project			
Almost always (vs. never)	–0.398*	0.008	0.750
Pretty often (vs. never)	–0.090	0.014	0.223
Once in a while (vs. never)	0.093*	0.002	–0.246
Having the teacher ask us what we know related to the new topic			
Almost always (vs. never)	–0.256*	0.005	0.300
Pretty often (vs. never)	0.044	0.003	–1.053*
Once in a while (vs. never)	0.085*	0.001	0.941*

(continued)

TABLE 5 (continued)

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Looking at the textbook while the teacher talks about it			
Almost always (vs. never)	–0.213*	0.060*	0.562
Pretty often (vs. never)	0.034	0.006	–0.321
Once in a while (vs. never)	0.208*	0.002	–0.641
Trying to solve an example related to the new topic			
Almost always (vs. never)	–0.016	0.003	0.890*
Pretty often (vs. never)	–0.013	0.001	–0.747*
Once in a while (vs. never)	0.042	0.004	–0.853

NOTE: Teacher instructional effect measures the effect of an instructional method on academic achievement, estimated in the presence of age, gender, mother's education, and father's education as statistically significant covariates. Variation in teacher instructional effect measures the extent to which teacher instructional effect varies across schools. School mean instructional effect measures whether the average degree to which teachers practice an instructional method in a school influences student academic achievement, estimated in the presence of school female enrollment (1 unit = 100 students) as the statistically significant covariate.
*$p < .05$.

TABLE 6: Different Instructional Methods of Beginning a New Topic in Mathematics on Student Performance in Measurement

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Having the teacher explain the rules and definitions			
Almost always (vs. never)	–0.003	0.010	0.089
Pretty often (vs. never)	0.035	0.008	–0.252
Once in a while (vs. never)	–0.049	0.055	0.262
Discussing a practical or story problem related to everyday life			
Almost always (vs. never)	–0.243*	0.018	1.222*
Pretty often (vs. never)	–0.007	0.006	–0.433
Once in a while (vs. never)	0.142*	0.008	–0.100
Working together in pairs or small groups on a problem project			
Almost always (vs. never)	–0.351*	0.002	0.358
Pretty often (vs. never)	–0.030	0.005	0.178
Once in a while (vs. never)	0.061	0.003	–0.151
Having the teacher ask us what we know related to the new topic			
Almost always (vs. never)	–0.092*	0.034	0.102
Pretty often (vs. never)	0.003	0.001	–0.708*
Once in a while (vs. never)	–0.019	0.001	0.690*

(continued)

TABLE 6 (continued)

Instructional Method	*Teacher Instructional Effect*	*Variation in Teacher Instructional Effect*	*School Mean Instructional Effect*
Looking at the textbook while the teacher talks about it			
Almost always (vs. never)	–0.047	0.017	0.422
Pretty often (vs. never)	–0.045	0.001	–0.348
Once in a while (vs. never)	0.088*	0.001	–0.399
Trying to solve an example related to the new topic			
Almost always (vs. never)	0.030	0.016	0.821*
Pretty often (vs. never)	–0.058	0.003	–0.356
Once in a while (vs. never)	0.074	0.007	–1.251*

NOTE: Teacher instructional effect measures the effect of an instructional method on academic achievement, estimated in the presence of age, gender, immigration status, and mother's education as statistically significant covariates. Variation in teacher instructional effect measures the extent to which teacher instructional effect varies across schools. School mean instructional effect measures whether the average degree to which teachers practice an instructional method in a school influences student academic achievement, estimated in the presence of school mean mother's education and school female enrollment (one unit = 100 students) as statistically significant covariates.
*$p < .05$.

academic achievement in other mathematical areas to those in the case of overall achievement in mathematics. For the sake of space, we provided only a summary of results across Tables 2 through 6, with an emphasis on similarities and differences in results.

Results in Tables 2 through 6 (in relation to those in Table 1) showed that having the teacher explain the rules and definitions as a way to begin a new topic in mathematics had instructional effects on student mathematics performance in geometry only. Using this instructional method once in a while had a positive effect, whereas using it pretty often had a negative effect. There was variation in teacher instructional effect across schools in data analysis and fraction. School mean instructional effect (over and above teacher instructional effect) turned out to be null.

Discussing a practical or story problem related to everyday life consistently showed a positive effect on academic achievement across all mathematical areas when mathematics teachers used it once in a while to introduce new topics in mathematics but a negative effect across all areas when teachers relied on it almost always to introduce new topics. When teachers used this instructional method more often, the instructional effect was either negative (in overall achievement, data analysis, fraction, and geometry) or null (in algebra and measurement). There was no variation in teacher instructional effect across schools. Positive school mean instructional effects occurred in geometry and measurement.

Working together in pairs or small groups on a problem project had a negative effect on student mathematics performance in each and every mathematical area when mathematics teachers relied on it almost always to introduce new topics in mathematics. Using this instructional method once in a while had a positive effect in algebra and geometry but a negative effect in fraction. There was no variation in teacher instructional effect across schools. We did not find any school mean instructional effect over and above teacher instructional effect in any mathematical area.

Having the teacher ask students what they know related to the new topic as an instructional method to begin a new topic in mathematics showed a positive effect on academic achievement in five of six mathematical areas when mathematics teachers used it once in a while to introduce new topics in mathematics but a negative effect across all areas when teachers relied on it almost always to introduce new topics. This teacher instructional effect did not vary across schools. A positive school mean instructional effect over and above teacher instructional effect occurred in data analysis, geometry, and measurement (associated with using this instructional method once in a while), and a negative school mean instructional effect occurred across all areas (associated with using this instructional method more often).

Looking at the textbook while the teacher talks about it had either negative (in overall mathematics, data analysis, fraction, and geometry) or null (in algebra and measurement) effects when mathematics teachers used it almost always as an instructional method to begin a new topic in mathematics. This instructional method had either positive (in overall mathematics, data analysis, and fraction) or null (in algebra, geometry, and measurement) effects when teachers used it more often. It had either positive (in geometry and measurement) or null (in overall mathematics, algebra, data analysis, and fraction) effects when teachers used it once in a while. The teacher instructional effect also varied across schools in fraction and geometry. We did not find any school mean instructional effect over and above teacher instructional effect.

The instructional method to begin a new topic in mathematics in which teachers try to solve an example related to the new topic did not demonstrate teacher instructional effect in any mathematical area but had a consistent positive school mean instructional effect over and above teacher instructional effect in all mathematical areas (associated with using this instructional method almost always). There were also two negative school mean instructional effects in geometry and measurement associated with using this instructional method more often. Teacher instructional effect did not vary across schools.

DISCUSSION

Our analysis of the effects of different instructional methods to begin a new topic in mathematics on student mathematics performance has important implications for classroom mathematics instruction. Several patterns emerged from our analysis that functioned to identify the most effective and the least effective instructional methods to introduce new topics in mathematics, as far as student mathematics performance was concerned. Overall, the way mathematics teachers introduced a new topic in mathematics contributed to the well-being of students in the learning of mathematics.

HAVING THE TEACHER EXPLAIN THE RULES AND DEFINITIONS

This instructional method to begin a new topic in mathematics stood out on two counts. First, there was little impact of this instructional method on student performance in almost all mathematical areas. Second, the average extent to which teachers practiced this instructional method in a school had no impact on student performance in any mathematical area. We suggest that these findings are a good indication of the ineffectiveness of having mathematics teachers simply explain the rules and definitions as a way to introduce a new topic in mathematics.

The failure of having the teacher explain the rules and definitions as an instructional method to begin a new topic in mathematics comes as little surprise to us. This instructional method essentially reflects the traditional teacher-centered instructional format, the very source of discontentment among mathematics educators that has promoted most educational reforms concerning mathematics instruction. Far away from the essence of any theoretical perspective as reviewed earlier, students are least likely to engage in learning activities under such an instructional format.

DISCUSSING A PRACTICAL OR STORY PROBLEM RELATED TO EVERYDAY LIFE

This instructional method to begin a new topic in mathematics demonstrated two unique properties. First, when mathematics teachers relied on this instructional method almost always, student performance was negatively affected in all mathematical areas, whereas when mathematics teachers used this instructional method once in a while, student performance was positively affected in all mathematical areas. Therefore, the overuse of discussing a practical or story problem related to everyday life as an instructional method to introduce new topics in mathematics turned out to be harmful to student mathematics performance.

Many mathematics educators advocate the use of everyday life contexts in mathematics instruction (e.g., Lesh and Lamon 1992). However, the TIMSS 1999 video study has revealed that mathematics teachers in most high-achieving countries present mathematics problems in symbols rather than real-life contexts (see Hiebert et al. 2003). In Japan, for example, 89% of problems are presented in mathematical symbols, compared with 9% in real-life contexts. An appreciation of the relationship between school mathematics and everyday life may be important to student affective rather than cognitive well-being in mathematics. The TIMSS video study helps explain the negative overuse effect of discussing a practical or story problem related to everyday life as an instructional method to introduce new topics in mathematics.

The second unique property is the “contradiction” between negative teacher instructional effects and positive school mean instructional effects (associated with using this instructional method almost always to introduce new topics in mathematics). This scenario actually indicates that students performed worse in schools where more mathematics teachers practiced this instructional method. Again, the pattern fits well into the results of the TIMSS video study. Overall, we concluded that the occasional use of this instructional method benefited student mathematics performance but that the frequent use of this instructional method harmed student mathematics performance.

WORKING TOGETHER IN PAIRS OR SMALL GROUPS ON A PROBLEM PROJECT

The highlight of findings regarding working together in pairs or small groups on a problem project is that using this instructional method almost always to begin a new topic in mathematics harmed student performance in all mathematical areas. Although both constructivist theories and peer-learning theories argue for the effectiveness of this instructional method (see Hoffman 2002; Serafino and Cicchelli 2003), the results of our analysis suggested that it might not be suitable for introducing new topics in mathematics. We suspect that meaningful cooperative learning is difficult to establish when students face new, unfamiliar topics in mathematics.

The power of peer mentoring or peer learning is limited when all individuals in a group are attempting to grasp for the first time the same new mathematical concepts, principles, or procedures. This could explain why we found that even occasional use of this instructional method was not as fruitful to student mathematics performance as, say, the instructional method of discussing a practical or story problem related to everyday life. Overall, we caution mathematics teachers when they consider cooperative learning as an instructional method to introduce new topics in mathematics.

HAVING THE TEACHER ASK STUDENTS WHAT THEY KNOW RELATED TO THE NEW TOPIC

Two findings are unique about this instructional method as a way to begin a new topic in mathematics. First, when mathematics teachers relied on this instructional method almost always, student performance was negatively affected in all mathematical areas, whereas when mathematics teachers used this instructional method once in a while, student performance was positively affected in almost all mathematical areas. The overuse of this instructional method to introduce new topics in mathematics was harmful to student mathematics performance, similar to the case of discussing a practical or story problem related to everyday life.

The second unique finding is the negative school mean instructional effect across all mathematical areas (associated with using this instructional method to introduce new topics in mathematics pretty often). Linking this finding with the lack of teacher instructional effect, we concluded that this instructional method was a very common practice to begin new topics in mathematics among schools, even though its negative effect on student mathematics performance was also common across schools. We suspect that the “delicate” part for success is whether meaningful links can be established between prior and new knowledge when this instructional method is used to introduce new topics in mathematics. It obviously has a very limited function to help students understand new topics in mathematics if mathematics teachers do not go beyond a simple recall of a “launderer” list of relevant prior knowledge to establish a constructivist connection between prior and new knowledge.

LOOKING AT THE TEXTBOOK WHILE THE TEACHER TALKS ABOUT IT

The next inactive instructional method to having the teacher explain the rules and definitions is looking at the textbook while the teacher talks about it. Although the overall pattern suggested the negative overuse effect and the positive occasional use effect, similar to the case of discussing a practical or story problem related to everyday life, the former was much more inconsistent in effect than the latter. Looking at the textbook while the teacher talks about it is another typical teacher-centered instructional practice. The results of our analysis suggested that it was among the least effective as a way to introduce new topics in mathematics.

TRYING TO SOLVE AN EXAMPLE RELATED TO THE NEW TOPIC

This instructional method demonstrated a positive school mean instructional effect on student performance in each and every mathematical area, the only positive instructional effect on student mathematics performance associated with frequent use among all instructional methods. Linking this consistent finding with the consistent lack of teacher instructional effect, we concluded that student performance in all mathematical areas was positively affected in most schools in which mathematics teachers practiced this instructional method almost always. Therefore, trying to solve an example related to the new topic as an instructional method to begin a new topic in mathematics was an effective way to enhance student mathematics performance. We suggest that adopting a problem-solving approach to begin a new topic in mathematics is likely to engage students in learning activities.

Both constructivist and Vygotskian perspectives emphasize the importance of problems in the learning of mathematics. The National Council of Teachers of Mathematics (1989, 2000) has adequately reflected this notion by emphasizing the critical role of problem solving in mathematics education. If appropriate mathematical problems are chosen as examples, they create correct perceptions and outline correct procedures among students about a new topic to be learned, which helps later on with the actual understanding of the new topic. We note that the positive school mean instructional effect corresponded to a considerably frequent use (almost always). Because it stood out in contrast with the lack of any instructional effect associated with the categories of pretty often and once in a while, the problem-solving approach to begin a new topic in mathematics needs to be practiced constantly.

CONCLUSION

We identified trying to solve an example related to the new topic as the most effective instructional method to begin a new topic in mathematics and the two teacher-centered instructional methods (having the teacher explain the rules and definitions and looking at the textbook while the teacher talks about it) as the least effective. We suggest caution in using discussion of a practical or story problem related to everyday life as an instructional method to introduce new topics in mathematics in consideration of the balance between symbolic and realistic contexts for the learning of mathematics. The negative effects displayed by some instructional methods when used excessively (discussing a practical or story problem related to everyday life, working together in pairs or small groups on a problem project, and having the teacher ask what the students know related to the new topic) call mathematics teachers to carefully study each new topic in mathematics and make a reasoned decision regarding which instructional method best suits the new topic (rather than relying on one instructional method). Modification of instructional methods for different needs, interests, and abilities of all students is another response to those negative instructional effects (see Lefrancois 2002; Secada 1992).

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